TEACHING WITH A 'CRITICAL FEATURE' OF AN 'OBJECT OF LEARNING' IN FOCUS

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Four teachers and a researcher participated in a learning study. In this paper we provide a brief overview of learning study and then describe the problem we engaged with during our study. We focus on the second iteration of the lesson as it was central to the emergence of the critical feature. We then move to the fourth lesson since this was the only lesson where the critical feature for the object of learning was in focus for the teacher. We also reflect briefly on the learning gains made by the learners as well as on shifts in teachers' thinking as a result of participating in this study.

INTRODUCTION

Improving something as complex and culturally embedded as teaching requires the efforts of all the players, including students, parents, and politicians. **But teachers must be the primary driving force behind change.** They are best positioned to understand the problems that students face and to generate possible solutions. (Stigler & Hiebert, 1999, p. 135)

In this paper we describe how through participation in a learning study we began to reflect critically on our lessons and started to become the 'driving force behind change' as alluded to by Stigler and Hiebert (ibid.), and this with respect to our teaching practice. Mampotse and Taona have been teaching secondary school mathematics for many years and find that learners have difficulty in understanding the notion of function. More specifically, they found that Grade 10 learners were unable to distinguish between different classes of functions given its algebraic representation e.g. which function represents a parabola: $y = 2^x$ or $y = x^2$?

In this paper we illustrate this difficulty and engage in discussion of how the teachers gradually shifted their practice to assist their learners in overcoming this difficulty. We commence this paper by providing a brief overview of what a learning study entails and in doing this we explain the concepts *object of learning* and *critical feature*.

LEARNING STUDY – A BRIEF OVERVIEW

Marton and Pang (2006) characterise a learning study as a group of between two and six teachers working together to find a way of making it possible for learners to "appropriate a specific object of learning"(p. 194). They identify the object of learning11 as "a specific insight, skill, or capability that the students are expected to develop during a lesson or during a limited sequence of lessons" (Marton & Pang, 2006, p. 194).

In other words, it can be those ‘hard to teach’ or ‘hot spot’ topics and concepts. Once the object of learning has been identified, the implementation of learning study follows a cyclical process. The process is considered cyclical because a lesson is collaboratively planned, it is taught by one teacher in the group and the other teachers observe the lesson. At the end of the lesson the teachers discuss the lesson and make suggestions for possible changes to the planned lesson. The lesson is revised and taught by the next teacher in the learning study group[1]. This process is repeated until all teachers in the group have had a turn to teach. Each of the lessons commences with a pre-test and at the end of the lesson the learners are required to write a post-test. This form of testing provides one possible way in which to track learning gains, if any, as a result of the lesson taught.

This cyclical process commences with the teachers choosing a specific object of learning that is central to the curriculum and that is known to cause difficulty for learners. Once the object of learning is identified the group commences with the planning of the lesson(s) with “a special focus on making it possible for the students to appropriate the object of learning” (Marton & Pang, 2006, p. 195). In their planning of the lesson the teachers in the learning study group have as resources their own experiences and previous research. In the study being reported on in this paper, the teachers also had a researcher as a resource. During the planning of the lesson the teachers focus on how they will vary some aspects of the object of learning while others remain invariant during the sequence of the lesson. In the case of this study, what varies and what is invariant is derived from the principles inherent in variation theory[2].

The purpose of the iterative process is to provide opportunity for the emergence of a critical feature. It is important to note that a critical feature is not the same as the difficulty learners are experiencing in relation to the object of learning. Instead, it is the particular feature of the object of learning that the learners must be able to discern in order to experience the object of learning in a certain way. The critical feature cannot be drawn from the mathematics alone; it has to emerge in relation to the learners being taught. For example, the critical feature that emerged through this study was that in order for the learners to distinguish between the different classes of functions given its algebraic representation, their attention had to be drawn to the highest power of the independent variable.

OUR PROBLEM

Although we taught the section on functions to our Grade 10 learners during the second term they performed poorly on questions related to the function concept when tested in the mid-year examination. For the purpose of this paper we will focus on our learners' responses to the question[3]: In you answer script draw a neat graph of the equation f(x) = 2^x, show intercepts if any. The following extracts are typical learner responses to the above question:

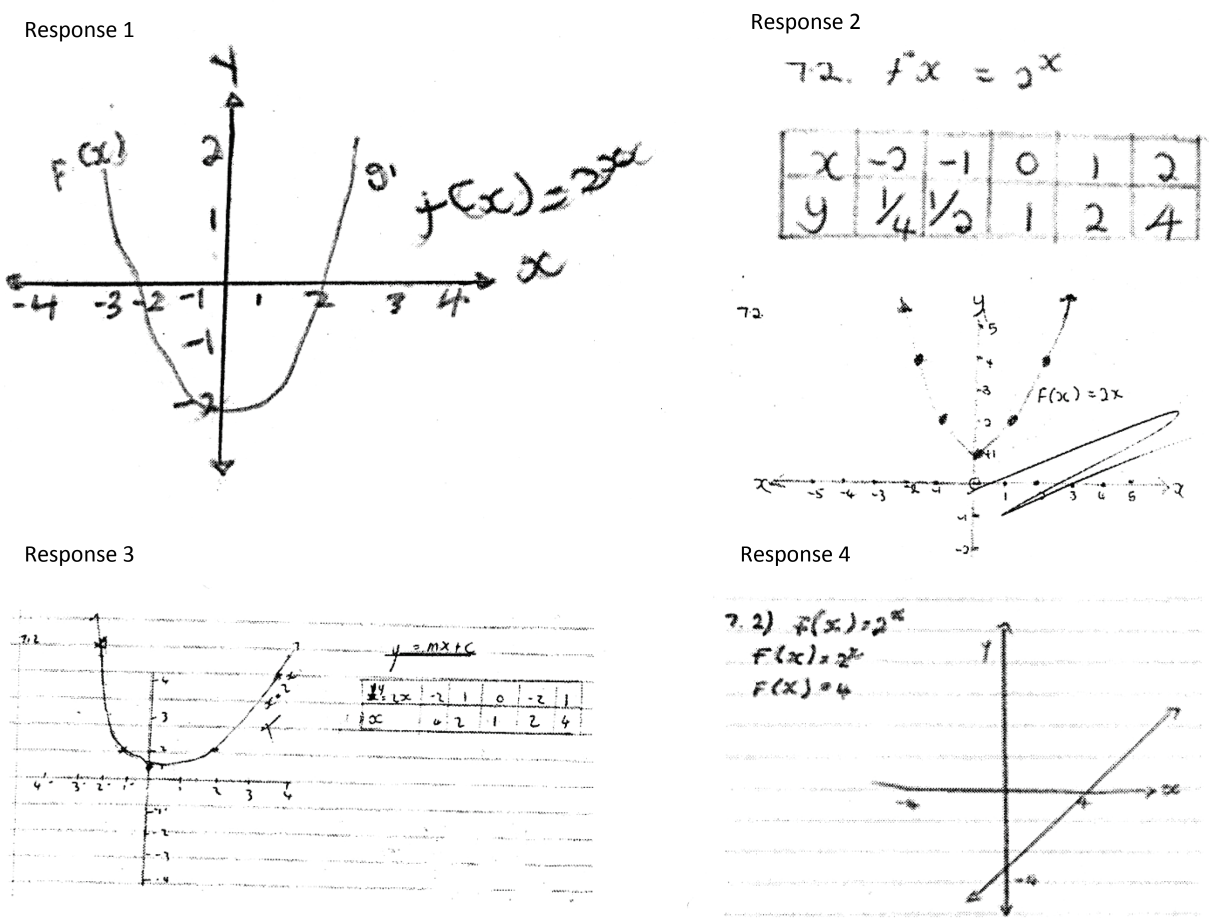

Figure 1: (Mis) recognition between multiple representations of a function

In Figure 1, responses 1, 2 and 3 suggest that the learners associated the function defined by $f(x) = 2^x$ with the quadratic function. They possibly (mis)recognised the structure and characteristics between 2^x and x^2.

In response 1, the learner did not show the derivation of the coordinates but drew a parabola with x-intercepts $(-2; 0)$ and $(2; 0)$ and y-intercept $(0; -2)$.

Interestingly, in response 2 learners were able to perform the correct arithmetic calculations after substituting various values of x into the given equation and represented these values correctly in a table, but plotted some of the points (where x is negative) incorrectly to form the shape of a parabola. In responses 1 and 2, it seems that the learners saw x^2 and 2^x as representing the same function. In response 4, the learners associated 2^x with a linear function which has a positive gradient and it is unclear as to how they arrived at the x-intercept $(4; 0)$ and y-intercept $(0; -4)$. In response 3, the learners signalled that they are interpreting $f(x) = 2^x$ as being linear by writing the general equation of a linear function $y = mx + c$. Furthermore, in completing the table of values the learners changed 2^x to 2x, which is then written in the cell to indicate x-values in the table of values. The rule used to relate input values with their respective output values is not clear and does not satisfy the equation defined by $y = 2x$. However, after representing the coordinates as a table of values the learners proceeded to draw a quadratic function. One can again assume that for these learners as well, $f(x) = 2^x$ represents the graph of a parabola.

Indeed there are indications that there are deeper difficulties for learners. Relatively simple algebraic forms appear to have little meaning for these learners. This raises some questions that are perhaps prior to multiple representations of functions: do learners see 2^x; x^2; and 2x as one and the same thing? What about $\frac{x}{2}$ or $\frac{2}{x}$? Do they understand the relationship between the variable and the constant in each instance? Do the learners recognise the mathematical operations implied in each of these instances and are they able to evaluate each of the expressions for a particular value of x? This is our point of departure as we start planning the first lesson of the learning study. The object of learning being: to enhance learners' ability to differentiate between the linear, quadratic, hyperbolic and exponential function given its algebraic representation. The learning study reported on in this paper had four iterations. For the purpose of this paper we will only focus on the second and fourth lessons since these two lessons are the critical lessons in this learning study cycle. The critical feature only emerged after the second lesson and was only brought into focus during the fourth lesson.

LESSON 2

To provide the context for lesson 2, we briefly describe what transpired in lesson 1. In lesson 1 the teacher worked across different classes of functions defined by: $y = 2x$; $y = x^2$; $y = \frac{1}{2}x$; $y = \frac{x}{2}$; $y = 2^x$ and $y = \frac{2}{x}$. In working across these functions he focused on substituting values for the input variable, performing the required mathematical manipulations as per the given equation to obtain the corresponding output value. The teacher then represented the input and its corresponding output value in a table. These examples constituted the range of examples that was used in each of the four lessons.

Absent from this lesson was the graphical representation of the functions. This absence was included in lesson 2. Lesson 2 commenced with Mampotse displaying the graphs of planned examples and then learners were required to identify the type of function represented and then give the algebraic form. As the Mampotse progressed from one example to another the learners' difficulty in correctly identifying the type of function and providing its algebraic form became obvious.

During the post-lesson discussion the researcher, Vasen, posed the following question to the teachers: what makes $y = 2x$ linear? Mathematically speaking $y = 2x$ is not linear simply because it is a polynomial function of the first degree, it is linear because it has a constant rate of change and $y = x^2$ is quadratic because the rate of change of the rate of change is a constant and not that it is a polynomial function of the second degree. In terms of relating the syntax of the algebraic representation of a function to its graph, focusing on the value of the exponent of the independent variable provides some criteria for learners to able to identify the class of function being represented by a given equation.

LESSON 4

To set the scene for lesson 4 we briefly explain what happed in lesson 3. In planning for lesson 3, the emphasis was on bringing the degree of the equation into focus. Although this was the focus of lesson 3, the critical feature was not in focus for the teacher as she went about teaching the lesson. The teacher focused on substituting input values to obtain output values and then represented these values in a table.

In lesson 4 the Taona introduced two functions in their algebraic form (e.g. $y = 2x$ and $y = \frac{1}{2}x$) and asked learners to compare the equations and say what is the same and what is different. It is through this process that he focused his learners' attention on the power of x. This was the first lesson where learners were provided with some tools by which to identify the class of function being represented by a particular equation.

LEARNING GAINS

After the emergence of the critical feature, Mampotse went back and re-taught the lesson to her own learners. Her learners were the learners that were taught in the first lesson of this learning study cycle. This now meant that two of the four groups of learners were exposed to the critical feature that emerged during the post-lesson 2 discussion. A few weeks after the last lesson was taught all the learners that participated in this study were required to write a delayed post-test. In this section we present data from this test.

Consider the following sets of bar charts (Figures 2 and 3) which illustrate the learners' performance in the post-test specifically for questions that required learners to identify and name a function given its equation. The learners who were taught in the first lesson in the learning study cycle are referred to as group 1 learners, the learners who were taught in the second lesson are referred to as group 2 learners, etc.

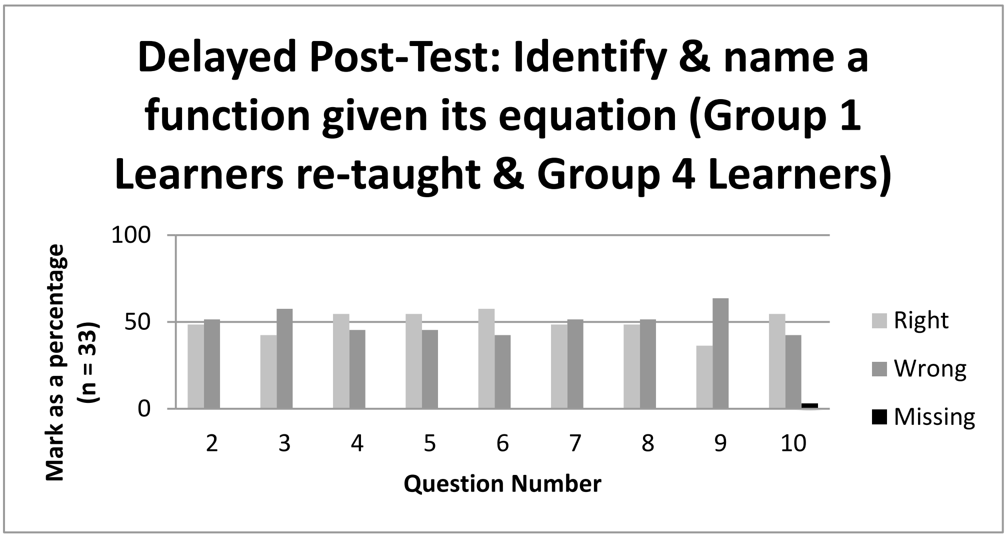

Figure 2: Post-test: identifying a function given its equation – Group 1 and Group 4 learners combined

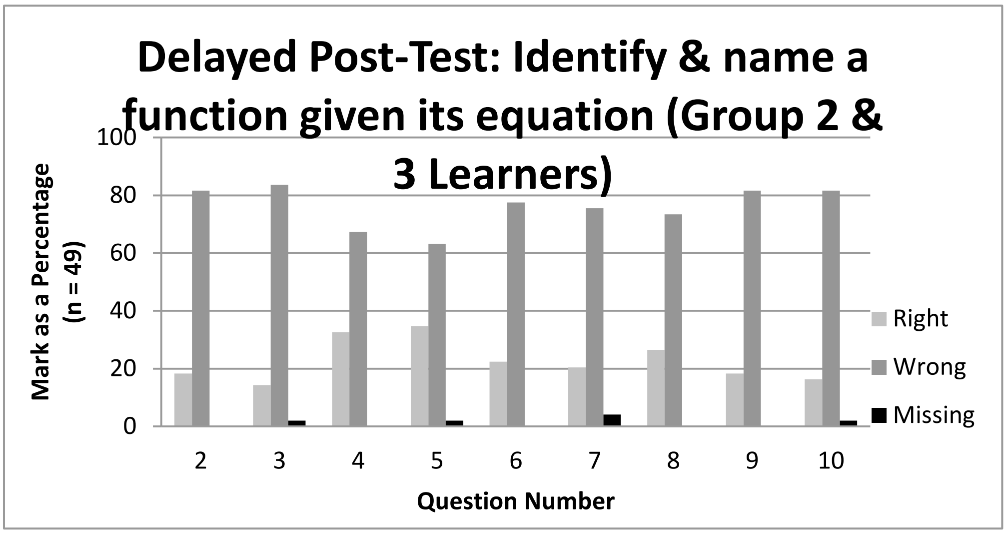

Figure 3: Post-test: identifying a function given its equation – Group 2 and Group 3 learners combined

Figure 2 illustrates group 1 and group 4 learners' performance on questions that required them to identify and name a function from its algebraic representation. It illustrates that, across the questions, between 40% and 60% of the learners were able to correctly identify the class of function being represented by the given equation. Figure 3 illustrates that, across all the questions, between 60% and 85% of the learners from groups 2 and 3 were **unable** to correctly identify and name the function given its algebraic representation. These two bar charts show that when the critical feature of the object of learning is in focus for the teacher (Figure 2) they are able to provide opportunities for learners to discern the object of learning and this is evidenced by a significant improvement in learners' performance in relation to questions pertaining to the object of learning.

SHIFTS IN TEACHERS' THINKING AS A RESULT OF PARTICIPATING IN THE STUDY

Prior to her participation in this study Mampotse saw the textbook as a resource in terms of it guiding her in the sequencing of her lessons and acting as a source for examples. She saw the role of examples as merely providing learners with exposure to different conditions under which a concept has to be recruited when solving a problem. In responding to what informed her choice of examples prior to the study, she referred to the level of the learners, where 'level' refers to how well learners perform in mathematics assessments. For learners who perform well, Mampotse selects more complex examples from the textbook and for weaker learners she selects less complex examples. She also indicated that previously she selected examples that ranged from the simplest to the most difficult and that she chose the examples randomly from the textbook.

After her participation in this study Mampotse indicated that: "this learning study has helped me in discovering that the object of learning should be the driving force for any lesson […] I have been made aware of the fact that examples do play an important / if not [a] major role in the planning and teaching of the lesson". She now believes that "the examples used should be such that they tend to reinforce the object of learning and are chosen in such a way that they make the learners discern the object of learning". She also sees value in working collaboratively and commented that the "community of practice should be able to periodically meet to iron out errors, misconceptions and problems encountered by teachers in the teaching of different mathematical topics". This comment reinforces that as a result of Mampotse participating in this study, she now attaches value to working with learner error or misconceptions as the point of departure in planning a lesson – we think about and plan a lesson (including the selection of examples) with an object of learning in mind.

Taona also indicated that he used the textbook to guide him with the content that had to be taught and that the textbook was a source from which to choose examples and he supplemented this with past exam papers. He also took into account the level of his learners in terms of their performance in the various assessments when choosing examples for use during the lesson and for homework purposes. He saw the role of examples as a means by which to explain the content that he had to teach, and after the lesson the examples given to learners as homework were for practice and for purposes of establishing for themselves the degree to which they understood the concepts taught.

After participating in the study, he indicated that: "it was an eye opener in as far as the relationship between planning and lesson execution is concerned". He went on to explain this as follows: "I have learnt that in planning one has to be clear of the steps to follow and the sequencing of activities.

Examples used should be in some chronology that builds up to the full attainment of the lesson objectives”. The chronology of the examples is the use of variation to structure and sequence of the examples so that the resulting example space brings the object of learning into focus. He also indicated that what he had learnt was that the “object of learning should always be the reference point in the whole process of teaching”. He now sees the need to ensure that when examples are selected they must be done with the object of learning in mind and that the examples should be carefully sequenced during the delivery of the lesson.

CONCLUSION

What this learning study managed to achieve was to place on the table and so bring into focus a critical feature that would facilitate an improvement in learners’ ability to identify and name different classes of functions, given their algebraic representation. It also demonstrates a practice in which teachers become deliberate about identifying learner difficulty and deliberately intervene to attend to the learner difficulty.

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